Practice Exam 2

Math 311
October 10, 2012 **Name:** ____________________

Read all of the following information before starting the exam:

- This test has 5 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I may take points off for rambling and for incorrect or illegible work.
- If your solution is long or complicated, you're over-thinking it or on the wrong track.
- A few vector identities which may be useful to you are:

$$\nabla(\phi_1\phi_2) = \phi_1\nabla\phi_2 + \phi_2\nabla\phi_1$$
$$\nabla \cdot \phi\mathbf{F} = \phi\nabla\cdot\mathbf{F} + \mathbf{F}\cdot\nabla\phi$$
$$\nabla \times \phi\mathbf{F} = \phi\nabla\times\mathbf{F} + \nabla\phi\times\mathbf{F}$$
$$\nabla f(\phi) = \frac{df}{d\phi}\nabla\phi$$
$$\nabla\cdot(\mathbf{R}-\mathbf{A}) = 3$$
$$\nabla\times(\mathbf{R}-\mathbf{A}) = \mathbf{0}$$
$$\nabla(|\mathbf{R}-\mathbf{A}|^n) = n|\mathbf{R}-\mathbf{A}|^{n-2}(\mathbf{R}-\mathbf{A})$$
$$\mathbf{F}\cdot\nabla(\mathbf{R}-\mathbf{A}) = \mathbf{F}$$
$$\nabla(\mathbf{A}\cdot\mathbf{R}) = \mathbf{A}$$
$$\nabla\cdot(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot(\nabla\times\mathbf{F}) - \mathbf{F}\cdot(\nabla\times\mathbf{G})$$
$$\nabla\times(\mathbf{F}\times\mathbf{G}) = (\mathbf{G}\cdot\nabla)\mathbf{F} - (\mathbf{F}\cdot\nabla)\mathbf{G} + (\nabla\cdot\mathbf{G})\mathbf{F} - (\nabla\cdot\mathbf{F})\mathbf{G}$$
$$\nabla\times(\nabla\times\mathbf{F}) = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$$
$$\nabla(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F})$$
$$\nabla\times\nabla(\phi) = \mathbf{0}$$
$$\nabla\cdot(\nabla\times\mathbf{F}) = 0$$
$$\nabla\cdot(\nabla\phi_1\times\nabla\phi_2) = 0$$

1. (*20 points*) Show in cylindrical coordinates that

$$\mathbf{r} = \rho\hat{\mathbf{e}}_\rho + z\hat{\mathbf{e}}_z$$

Hint: You will need to express the Cartesian unit vectors in terms of the new coordinates.

2. (*20 points*) Derive the unit vectors $\hat{\mathbf{e}}_r$, $\hat{\mathbf{e}}_\theta$, and $\hat{\mathbf{e}}_\phi$ for the spherical coordinate system and show they are orthogonal

$$x = r\cos\theta\sin\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\phi$$

3. (*20 points*) Use vector identities to simplify the expression

$$\nabla \times [(\mathbf{A}\cdot\nabla)\mathbf{F} + \mathbf{A}\times(\nabla\times\mathbf{F})]$$

where $\mathbf{A}$ is a constant vector and $\mathbf{F}$ is a variable vector field.

4. (*20 points*) Evaluate the line integral over the straight segment C joining the point $P = (2,1,4)$ to $Q = (3,3,4)$

$$\int_C 3xy\,dx + 3\,dy + yz\,dz$$

5. (*20 points*) Determine the vector field $\mathbf{F}$ that satisfies the conditions

$$\nabla\times\mathbf{F} = x\mathbf{j}, \quad \nabla\cdot\mathbf{F} = 0, \quad \mathbf{F}(0,0,0) = 0$$